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13. ABSTRACT (Maximum 200 words)

In this technical report, we develop polynomial expressions for the coefficient of nonlinearity β for both seawater and fresh water. The seawater relations are for salinities of 30-40 ppt, temperatures of 0-40°C, and pressures of 1-800 bars absolute. The fresh water relations are for temperatures of 0-90°C and pressures of 1-1000 bars absolute. Also presented are polynomial expressions for the term $\Lambda \equiv \beta/(\rho c^5)^{1/2}$, which arises in nonlinear geometrical acoustics. The precision of the fits of the polynomials to the underlying data is better than 1 ppt in all cases. However, the uncertainty in the seawater β relation is ±3% for realistic ocean temperatures and pressures. An uncertainty estimate was not obtained for the fresh water β relation.

14. SUBJECT TERMS

acoustics	high pressure
coefficient of nonlinearity	high temperature
fresh water	seawater

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1. INTRODUCTION

In the study of nonlinear acoustic propagation through an inhomogeneous fluid such as seawater, it is helpful to have explicit relations for the acoustic properties of the fluid as a function of temperature, pressure, and composition. For both fresh water and seawater, several empirical relations give the sound speed c and the density ρ as functions of temperature, pressure, and, in the case of seawater, salinity S. We are, however, unaware of any similar relations for the coefficient of nonlinearity β. Endo [1] has tabulated β for seawater over a broad range of temperatures and pressures for salinities of 25, 30, 35, and 40 parts per thousand (ppt). For computer implementations of nonlinear propagation through seawater, however, relations with explicit temperature, pressure, and salinity dependence are preferable over tables of values. Endo's tabular values were useful in checking our calculations.

In this work three relations for β are developed: one for fresh water as a function of temperature and pressure, one for seawater with a salinity of 35 ppt as a function of temperature and pressure, and one for seawater as a function of temperature, pressure, and salinity. Also developed is a similar set of relations for the term $\Lambda \equiv \beta/(\rho c^5)^{1/2}$.

The reason for developing separate relations for Λ is that it arises as an important variable in nonlinear geometrical acoustics [2,3]. Under the assumptions of nonlinear geometrical acoustics, the equation governing the propagation of a progressive finite-amplitude acoustic wave in an inhomogeneous fluid may be converted to the equation governing the propagation of a similar plane wave in a homogeneous fluid. The conversions required are the following:

$$\overline{p} \equiv p \left(\frac{A}{A_0}\right)^{1/2} \left(\frac{\rho c}{\rho_0 c_0}\right)^{-1/2} \quad , \tag{1.1}$$

$$\overline{x} \equiv \int_{\ell_0}^{\ell} \frac{\Lambda}{\Lambda_0} \left(\frac{A}{A_0}\right)^{-1/2} d\ell' \quad , \tag{1.2}$$

where p is the acoustic pressure, $\overline{x}$ and $\overline{p}$ are, respectively, the equivalent plane wave propagation distance and pressure, A is the ray tube area, and ℓ is the ray path length. The subscript $_0$ indicates conditions at the source point. Equation (1.2) makes clear why an explicit relation for Λ as a function of temperature, pressure, and salinity is helpful.

2. OVERVIEW OF PROCEDURE

The coefficient of nonlinearity is defined as follows: $\beta = B/2A+1$, where A and B are the first and second coefficients of the Taylor series expansion of the total pressure P in terms of the relative density change. (In fresh water at room temperature and atmospheric pressure, β has a value of approximately 3.5.) Rudnick [4] has shown that B/A may be expressed in terms of quantities that are easily related to physical data. Use of the expression for B/A as shown in the paper by Beyer [5] gives

$$\beta = \rho c \left(\frac{\partial c}{\partial P}\right)_T + \frac{c\alpha T}{C_P}\left(\frac{\partial c}{\partial T}\right)_P + 1 \quad , \tag{2.1}$$

where the subscripts indicate constant conditions, and T, α, and C_P are, respectively, the absolute temperature, the coefficient of thermal expansion, and the specific heat at constant pressure. Note that the first and second terms in Eq. (2.1) contain different partial derivatives of the sound speed $c(P,T)$ evaluated at constant composition. The term that contains $(\partial c/\partial P)_T$ is hereinafter referred to as "term 1", whereas the term that contains $(\partial c/\partial T)_P$ is hereinafter referred to as "term 2". Rudnick noted that, while all parts make a significant contribution to β, term 1 is numerically larger than term 2 in all common fluids.

In our evaluation of β, published empirical relations were used as much as possible. When a derivative of an acoustical property was required, the derivative was obtained by differentiating the published empirical relation analytically, then evaluating the result numerically.

To the best of our knowledge, there is only one set of empirical relations for the acoustical properties of fresh water that is valid over a wide range of temperatures (0–100°C) and pressures (1–1000 bars absolute). The relations used for the sound speed and density of fresh water are from, respectively, Chen and Millero [6] and Chen *et al.* [7]. The estimates of the coefficient of thermal expansion were obtained using ρ from Chen *et al.* [7] in the expression $-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$. For C_P, on the other hand, we used a polynomial fit to a fresh water C_P data set [8].

The situation is slightly different for seawater: at least two published empirical relations that are valid over a wide range of temperatures (0–40°C) and pressures (1–800 bars absolute) exist for both the sound speed and the density. For the sound speed the choice is between the relation developed by Lovett [9] and that developed by Chen and Millero [10]. For the density the choice is between the expressions developed by Gebhart and Mollendorf [11] and those from Chen and Millero [12]. The choice for the coefficient of thermal expansion is somewhat dependent on the choice made for the density. The choice is either the evaluation of $-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{P,S}$ using ρ from Gebhart and Mollendorf [11] or the explicit relation for α given by Chen and Millero [12]. Our estimate of the specific heat at constant pressure in seawater C_P was obtained by summing the estimate of C_P at one atmosphere, which was calculated using a relation from Millero *et al.* [13], with the contribution from the isothermal integration of [14]

$$-T\int \frac{1}{\rho}\left(\alpha^2 + \left(\frac{\partial \alpha}{\partial T}\right)\right) dP \quad . \tag{2.2}$$

The estimates of α and ρ for Eq. (2.2) were obtained from the appropriate aforementioned references.

3. UNCERTAINTY IN ESTIMATES OF β IN SEAWATER

To help decide which empirical relations to use for c, ρ, and α in seawater and, more importantly, to estimate the uncertainty in the calculation of β, we numerically implemented all of the aforementioned empirical relations. The absolute difference in β due to using two different empirical relations was calculated for one property at a time. The difference in β caused by terms 1 and 2 was calculated separately to see which term in β is more sensitive.

Term 1, the term in β that contains $(\partial c/\partial P)_T$, was found to be the largest source of uncertainty. Estimates of the uncertainty were obtained by calculating term 1 using Lovett's relation for c [9] while holding all other relations constant. (For ρ and α, the previously referenced Chen and Millero relations were used.) This result was then subtracted from a calculation of term 1 obtained using Chen and Millero's relation for c [10]. The difference was calculated over a realistic range of temperatures and pressures, that is, a range of temperatures and pressures that might be found in an ocean: a temperature range of -4–$40°$ for pressures of 1 and 100 bars absolute, but a temperature range of only -4–$10°$ for pressures of 200, 300, ..., 800 bars absolute. Shown in Fig. 3.1 is a plot of the difference in term 1 for salinities of 30 and 40 ppt. (Solid lines are for salinity of 30 ppt, dashed for 40 ppt.) For these salinities, the difference is always less than 0.1, typically 0.05. For a salinity of 35 ppt (not shown), the absolute difference is always less than 0.05 and typically less than 0.025. For contrast, Fig. 3.2 shows a plot of the corresponding difference in term 2, which was calculated in an identical fashion. The magnitude of these differences is much smaller, typically less than 0.0025.

The uncertainities in β that stem from the different empirical relations for ρ and α were examined in the same manner as those for c. Both terms 1 and 2 were evaluated twice, once using the density relation from Gebhart and Mollendorf [11] and once using Chen and Millero's relation [12]. Because α is closely related to ρ, it was varied simultaneously. (In all cases, the relation used for c was that of Chen and Millero [10].) Plots of the differences generated by terms 1 and 2 are shown, respectively, in Figs. 3.3 and 3.4. The largest differences in β that result are in term 2, and the differences are always less than 0.01.

Finally, a check on the pressure correction term in C_P, Eq. (2.2), was made at zero salinity. Between 1 and 1000 bar, the pressure increment in C_P given by Eq. (2.2) deviated by around 10% from the values in Ref 8. Nevertheless, the corresponding uncertainity in β, through term 2, is only 1 part in 10^4.

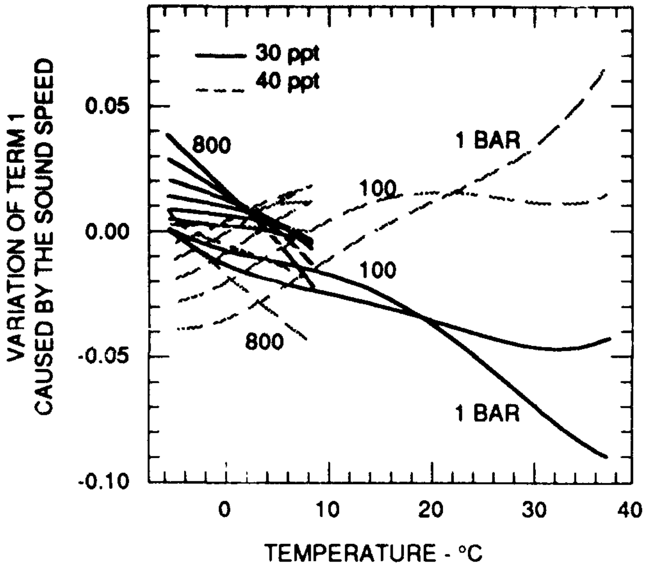

FIGURE 3.1
PLOTS OF THE ABSOLUTE DIFFERENCE IN
β versus TEMPERATURE FOR ABSOLUTE PRESSURES
OF 1, 100, 200, 300...800 BAR CAUSED BY USING DIFFERENT
EMPIRICAL RELATIONS FOR c IN TERM 1

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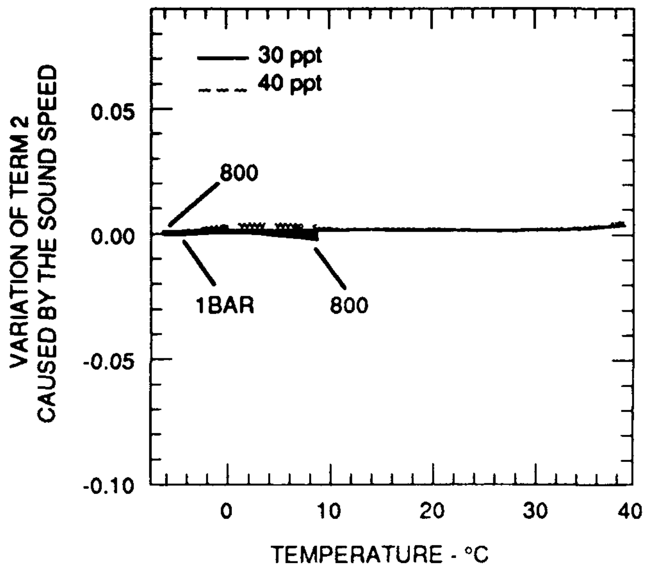

FIGURE 3.2
PLOTS OF THE ABSOLUTE DIFFERENCE IN
β versus TEMPERATURE FOR ABSOLUTE PRESSURES
OF 1, 100, 200, 300...800 BAR CAUSED BY USING DIFFERENT
EMPIRICAL RELATIONS FOR c IN TERM 2

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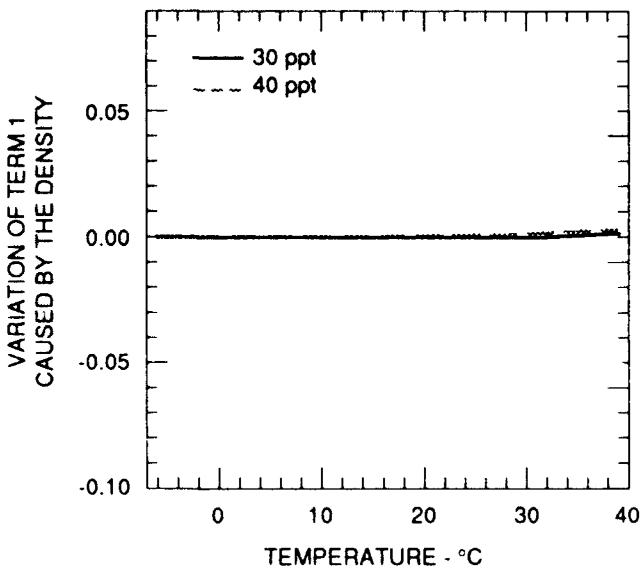

FIGURE 3.3
PLOTS OF THE ABSOLUTE DIFFERENCE IN
β versus TEMPERATURE FOR ABSOLUTE PRESSURES
OF 1, 100, 200, 300...800 BAR CAUSED BY USING DIFFERENT
EMPIRICAL RELATIONS FOR ρ AND α IN TERM 1

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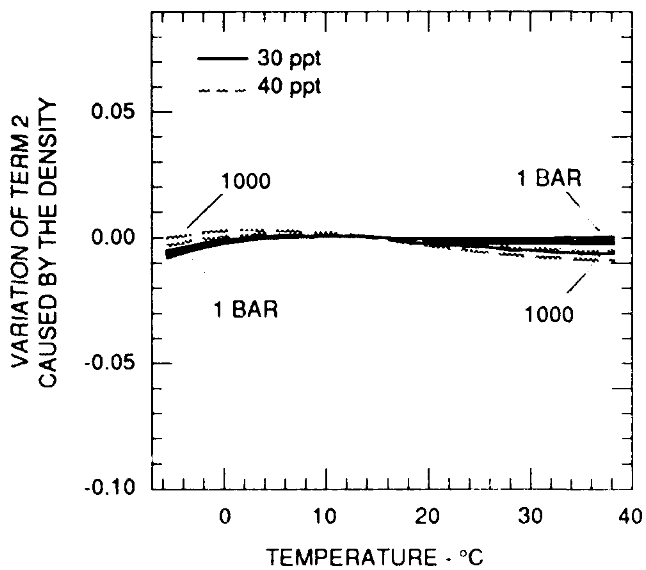

FIGURE 3.4
PLOTS OF THE ABSOLUTE DIFFERENCE IN
β versus TEMPERATURE FOR ABSOLUTE PRESSURES
OF 1, 100, 200, 300...800 BAR CAUSED BY USING DIFFERENT
EMPIRICAL RELATIONS FOR ρ AND α IN TERM 2

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Rather than embark on an investigation of the origin of the above cited differences, we simply chose to use the relations from Chen, Millero, and their co-workers over those of other workers. Our reasoning was uniformity of approach: Chen, Millero, and their co-workers have developed empirical relations not only for c, but also for ρ, α, and C_P. Thus, for seawater, the relations in Refs. 10 and 12 were used for c and ρ. Estimates of α were obtained using the explicit relation for α in seawater from Ref. 12. When using our polynomial expressions to calculate β, however, one must always bear in mind that the differences cited above are, generally speaking, the accuracy limits in our results.

4. RESULTS

4.1 PLOTS OF c, ρ, β, AND Λ

Plots of c, ρ, β, and Λ for fresh water are shown in Figs. 4.1–4.4 and for seawater in Figs. 4.5–4.8. For fresh water, the acoustic properties are plotted as functions of temperature for the absolute pressures of 1, 100, 200, 300, ..., 1000 bars. Because the fresh water C_P relation is valid up to only 90°C, our relations for β and Λ in fresh water must be used with caution above 90°C. The curves are therefore dashed in this region. In the case of seawater, Figs. 4.5–4.8, the salinity is held constant at 35 ppt. The acoustic properties are plotted as functions of temperature for absolute pressures of 1, 100, 200, 300, ..., 800 bars. Because the accuracy of the relations for c and ρ in seawater is uncertain below 0°C, the values for c and ρ below 0°C are dashed lines in this region. Similarly, our derived relations for β and Λ in seawater must be used with caution below 0°C, and they too are dashed lines in this region.

Some discussion of the plots is in order. The plots of the sound speed and density for both fresh water and seawater are as expected: the density decreases with rising temperature, but increases with rising pressure, whereas the sound speed increases with pressure and, up to around 70–80°, increases with temperature also. The plots of the coefficient of nonlinearity reveal a more complex temperature and pressure dependence. In seawater, β increases with pressure for the temperature range considered. However, in fresh water, the sign of the pressure derivative of β is temperature dependent. Below about 60°C, β increases monotonically with pressure, whereas above that temperature, β initially decreases with rising pressure. For both fresh water and seawater, the pressure effect on β tends to decrease with rising temperature. The variation in β near 0°C is about 25%, whereas at higher temperatures the variation decreases to about 5%. In contrast with β, the pressure effect on Λ increases with temperature for both fresh water and seawater. Near 0°C, the variation is about 5%, but by 40°C the variation has increased to about 25%, and (in fresh water) by 90°C the variation has increased to about 40%. Aside from a deviation at low pressure and temperature, the temperature and pressure dependences of Λ for both seawater and fresh water are generally opposite those of the sound speed c. This is reasonable given that $c^{5/2}$ appears in the denominator of Λ.

A comparison of the isobar plots of β for fresh water and seawater (Fig. 4.3 with Fig. 4.7) casts some doubt on the validity of our extrapolation of the underlying expressions for the acoustic properties below 0°C. Note that, in our fresh water β

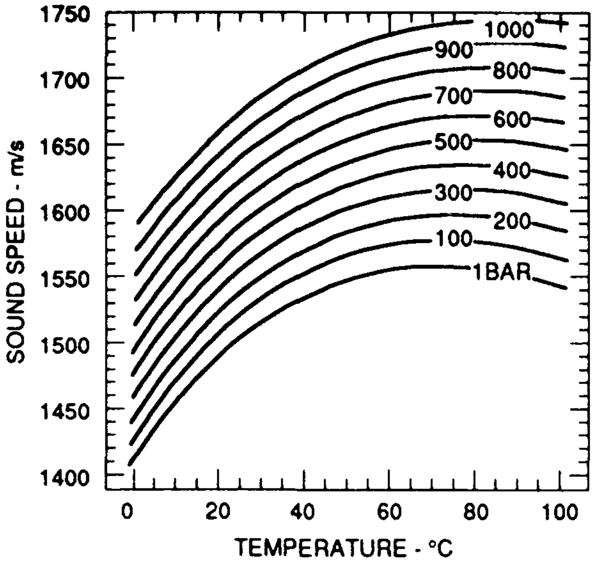

FIGURE 4.1
PLOTS OF SOUND SPEED versus TEMPERATURE
FOR VARIOUS ABSOLUTE PRESSURES (BAR)
IN FRESH WATER

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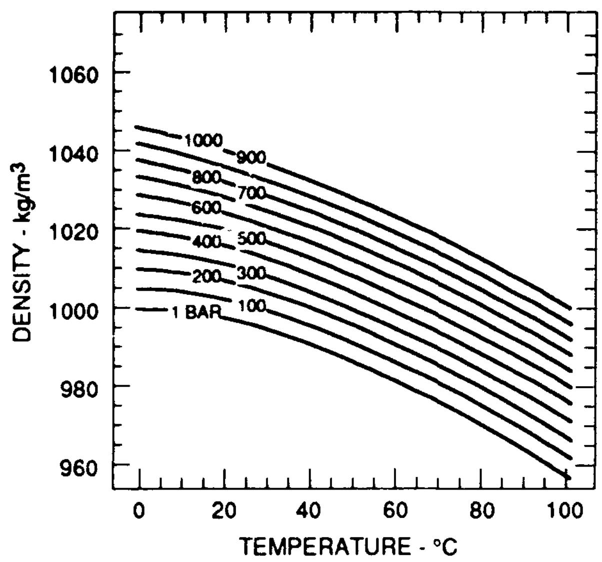

FIGURE 4.2
PLOTS OF DENSITY versus TEMPERATURE
FOR VARIOUS ABSOLUTE PRESSURES (BAR)
IN FRESH WATER

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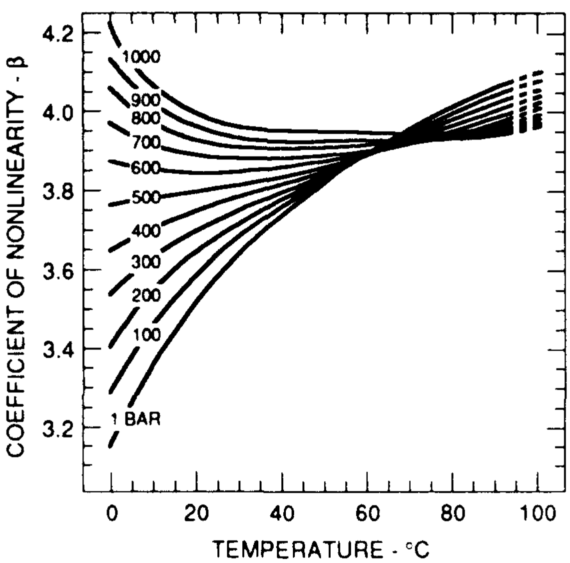

FIGURE 4.3
PLOTS OF β versus TEMPERATURE
FOR VARIOUS ABSOLUTE PRESSURES (BAR)
IN FRESH WATER

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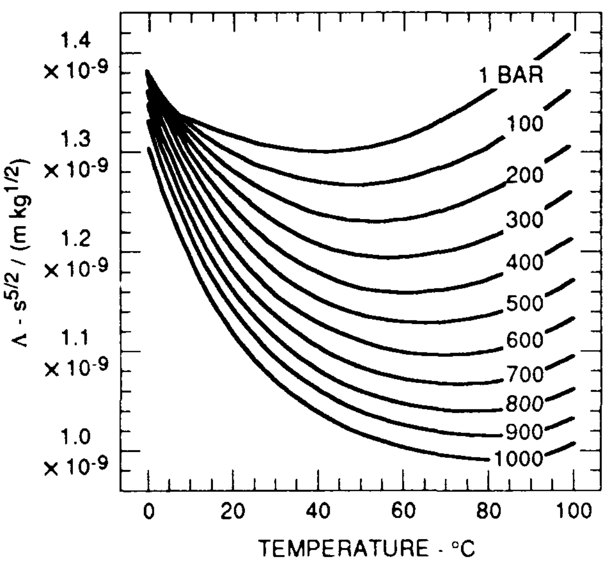

FIGURE 4.4
PLOTS OF Λ versus TEMPERATURE
FOR VARIOUS ABSOLUTE PRESSURES (BAR)
IN FRESH WATER

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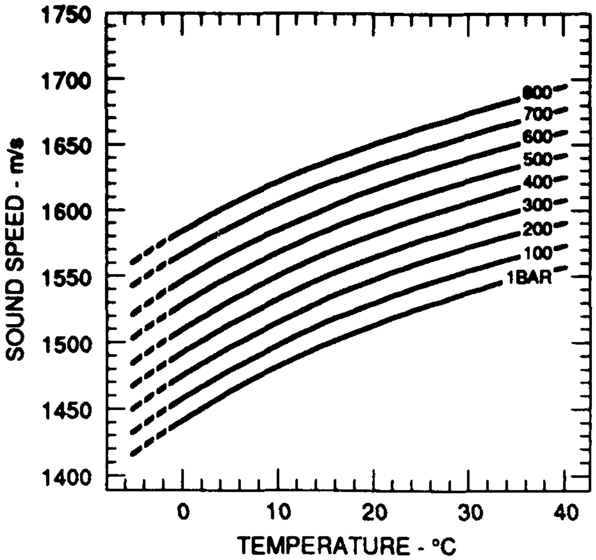

FIGURE 4.5
PLOTS OF SOUND SPEED versus TEMPERATURE
FOR VARIOUS ABSOLUTE PRESSURES (BAR)
IN SEAWATER WITH A SALINITY OF 35 ppt [10]

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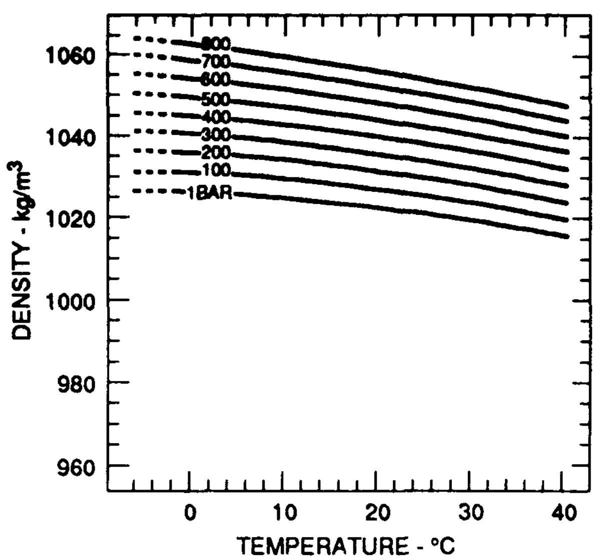

FIGURE 4.6
PLOTS OF DENSITY versus TEMPERATURE
FOR VARIOUS ABSOLUTE PRESSURES (BAR)
IN SEAWATER WITH A SALINITY OF 35 ppt [12]

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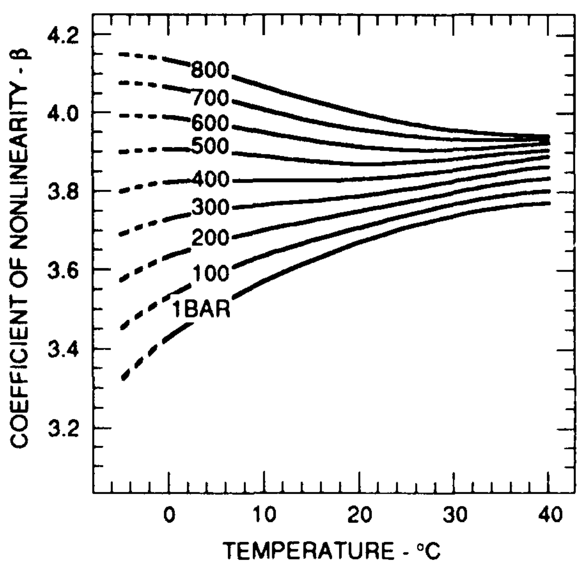

FIGURE 4.7
PLOTS OF β versus TEMPERATURE
FOR VARIOUS ABSOLUTE PRESSURES (BAR)
IN SEAWATER WITH A SALINITY OF 35 ppt

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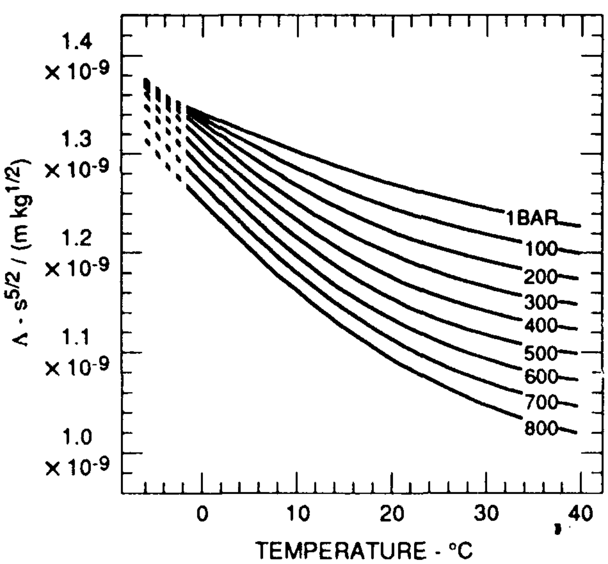

FIGURE 4.8
PLOTS OF Λ versus TEMPERATURE
FOR VARIOUS ABSOLUTE PRESSURES (BAR)
IN SEAWATER WITH A SALINITY OF 35 ppt

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results, the β curves diverge rapidly as they approach the freezing point, whereas in our seawater β results, the β curves do not exhibit a similar divergence below 0°C. The reason may be our unjustified extrapolation of the underlying expressions for c, ρ, α, and C_P in seawater. Although these relations are not stated as being accurate below 0°C, they are the best available. Because we believe that having a expression for β with an uncertain accuracy below 0°C is preferable over no expression at all, we extrapolated the underlying expressions to develop our relation for β.

4.2 POLYNOMIAL EXPRESSIONS FOR β AND Λ

Our polynomial expressions for β and Λ are now presented. The order of our polynomial fit was chosen to be similar to that used in the empirical fit to the dominant underlying acoustic property, the sound speed c. The sound speed relation given by Chen and Millero [7] is fifth order in temperature, third order in pressure, and second order in salinity. The order of the salinity terms was, however, reduced from second to first order because the sound speed relation given by Chen and Millero [7] is valid for a broader range of salinities than is required (5–40 ppt rather than our 30–40 ppt). The resulting fit of the polynomials to the original data is more than adequate. The quality of the fits of the polynomials to the original data was assessed through direct comparison, and the comparison was performed over the claimed ranges of temperatures, pressures, and salinities. The difference between the original data and the fit is always far less than the previously discussed uncertainty in the overall calculation.

Our polynomial expressions for β were obtained as follows. For seawater, Eq. (2.1) was evaluated over the following range: temperature −4 to 40°C, pressure 1–800 bars absolute, and salinity 30–40 ppt. The resulting data points were then fitted to a polynomial expression that is fifth order in temperature, third order in pressure, and linear in salinity. The polynomial expression is

$$\beta = \sum_{l,m,n} B_{lmn}(S - 35)^l P^m T^n \quad , \tag{4.1}$$

where the salinity S is in ppt, the temperature T is in °C, and the pressure P is bars gauge, that is, relative to 1 atm = 1.01325 bar.[1] The coefficients B_{lmn} are listed in Table 4.1. The maximum absolute error in the fit is 0.0015, and the typical error is about half that. (Check value: For a salinity of 32.5 ppt, a pressure of 100 bar gauge, and a temperature of 15°C, β is 3.672556.)

The actual salinity of the ocean water column is frequently unknown. In its place, people often assume a salinity of 35 ppt because this salinity is so common in the

[1] This is the convention used by most authors, including Chen, Millero, and their co-workers, for presenting empirical fits to seawater and fresh water property data.

TABLE 4.1
Coefficients in polynomial expression for β

l	m	n	B^{0}_{mn}	B_{lmn}	B^{35}_{mn}
0	0	0	+3.122127	+3.44166	+3.44223
0	0	1	+0.0262768	+0.0176224	+0.0176264
0	0	2	−0.000439164	−0.000478735	−0.00047893
0	0	3	+0.00000520032	+0.0000129976	+0.0000130006
0	0	4	-3.07095×10^{-8}	-1.85398×10^{-7}	-1.85429×10^{-7}
0	0	5	$+6.12344 \times 10^{-11}$	$+3.36832 \times 10^{-10}$	$+3.37367 \times 10^{-10}$
0	1	0	+0.00139138	+0.00104125	+0.0010417
0	1	1	−0.000051128	−0.0000457645	−0.0000457763
0	1	2	$+7.40662 \times 10^{-7}$	$+5.84691 \times 10^{-7}$	$+5.85248 \times 10^{-7}$
0	1	3	-6.41226×10^{-9}	$+5.16276 \times 10^{-9}$	$+5.14004 \times 10^{-9}$
0	1	4	$+2.53274 \times 10^{-11}$	-3.84942×10^{-11}	-3.82208×10^{-11}
0	2	0	-2.15517×10^{-7}	-1.69183×10^{-7}	-1.69416×10^{-7}
0	2	1	$+3.45228 \times 10^{-9}$	$+1.69021 \times 10^{-8}$	$+1.69053 \times 10^{-8}$
0	2	2	$+1.44653 \times 10^{-10}$	-3.37325×10^{-10}	-3.37744×10^{-10}
0	2	3	-2.82686×10^{-12}	-3.82096×10^{-12}	-3.79902×10^{-12}
0	2	4	$+1.26217 \times 10^{-14}$	$+4.52996 \times 10^{-14}$	$+4.49897 \times 10^{-14}$
0	3	0	-1.12614×10^{-10}	-6.58146×10^{-11}	-6.57756×10^{-11}
0	3	1	$+4.51141 \times 10^{-12}$	$+6.60716 \times 10^{-12}$	$+6.60637 \times 10^{-12}$
0	3	2	-2.91434×10^{-14}	-1.63621×10^{-13}	-1.63501×10^{-13}
1	0	0		+0.00833572	
1	0	1		−0.000232407	
1	0	2		+0.00000107536	
1	1	0		−0.0000115765	
1	1	1		$+7.2387 \times 10^{-9}$	
1	1	2		$+1.0786 \times 10^{-8}$	
1	2	0		$+3.2689 \times 10^{-9}$	
1	2	1		$+4.6156 \times 10^{-10}$	
1	2	2		1.86714×10^{-11}	

world's oceans. For these reasons, a less complicated relation for β for the salinity of 35 ppt was also developed,

$$\beta^{35} = \sum_{m,n} B^{35}_{mn} P^m T^n \quad . \tag{4.2}$$

Valid over the same range of temperatures and pressures, its coefficients are denoted B^{35}_{mn} and are listed in Table 4.1. The maximum absolute error in the fit is 0.0001. (Check value: For a pressure of 98.98675 bar gauge and a temperature of 15°C, β^{35} is 3.683292.)

For fresh water, Eq. (2.1) was evaluated over the temperature range 0–100°C and the pressure range 1–1000 bars absolute. A polynomial fit of the same order in temperature and pressure as above was then obtained,

$$\beta^{0} = \sum_{m,n} B^{0}_{mn} P^m T^n \quad . \tag{4.3}$$

The coefficients B^0_{mn} are also listed in Table 4.1, and the maximum absolute error of the fit is 0.0005. (Check value: For a pressure of 98.98675 bar gauge and a temperature of 15°C, β^0 is 3.508381.)

Polynomial fits for $\Lambda \equiv \beta/(\rho c^5)^{1/2}$ that are valid over the same ranges of temperatures and pressures for seawater with salinities of 30–40 ppt, seawater with a salinity of 35 ppt, and fresh water were developed as above. The relations are shown in Eqs. (4.4)–(4.6), and the coefficients, which are denoted, respectively, L_{lmn}, L^{35}_{lmn}, and L^0_{mn}, are listed in Table 4.3:

$$\Lambda = \left(\sum_{l,m,n} L_{lmn}(S-35)^l P^m T^n \right) \times 10^{-9} \quad \mathrm{s}^{5/2}\mathrm{m}^{-1}\mathrm{kg}^{-1/2} \quad , \tag{4.4}$$

$$\Lambda^{35} = \left(\sum_{m,n} L^{35}_{mn} P^m T^n \right) \times 10^{-9} \quad \mathrm{s}^{5/2}\mathrm{m}^{-1}\mathrm{kg}^{-1/2} \quad , \tag{4.5}$$

$$\Lambda^{0} = \left(\sum_{m,n} L^{0}_{mn} P^m T^n \right) \times 10^{-9} \quad \mathrm{s}^{5/2}\mathrm{m}^{-1}\mathrm{kg}^{-1/2} \quad . \tag{4.6}$$

The maximum absolute errors in these fits are, respectively, 0.0004×10^{-9}, 0.0001×10^{-9}, and 0.0015×10^{-9}. (Check values: For a salinity of 32.5 ppt, a pressure of 100 bar gauge, and a temperature of 15°C, Λ is 1.270685×10^{-9}. For a pressure of 98.98675 bar gauge and a temperature of 15°C, Λ^{35} is 1.267513×10^{-9} and Λ^0 is 1.309151×10^{-9}.)

TABLE 4.2
Coefficients in polynomial expression for Λ.

l	m	n	L^0_{mn}	L_{lmn}	L^{35}_{mn}
0	0	0	$+1.340649$	$+1.34255$	$+1.34282$
0	0	1	-0.00123676	-0.00369707	-0.00370087
0	0	2	-0.0000219506	-0.0000454961	-0.0000454702
0	0	3	$+7.51912 \times 10^{-7}$	$+0.00000411388$	$+0.0000041155$
0	0	4	-2.82344×10^{-9}	-7.30211×10^{-8}	-7.30791×10^{-8}
0	0	5	-5.22167×10^{-12}	$+2.49154 \times 10^{-10}$	$+2.49717 \times 10^{-10}$
0	1	0	$+0.000197704$	-2.88818×10^{-7}	-4.62069×10^{-7}
0	1	1	-0.0000256345	-0.0000191962	-0.0000191921
0	1	2	$+4.70095 \times 10^{-7}$	$+4.28444 \times 10^{-7}$	$+4.28422 \times 10^{-7}$
0	1	3	-4.83161×10^{-9}	-4.12041×10^{-9}	-4.1215×10^{-9}
0	1	4	$+1.90758 \times 10^{-11}$	$+4.36867 \times 10^{-11}$	$+4.37169 \times 10^{-11}$
0	2	0	-2.69563×10^{-7}	-1.43851×10^{-7}	-1.43841×10^{-7}
0	2	1	$+1.46666 \times 10^{-8}$	$+1.65626 \times 10^{-8}$	$+1.65666 \times 10^{-8}$
0	2	2	-1.99597×10^{-10}	-3.94804×10^{-10}	-3.95058×10^{-10}
0	2	3	$+1.59563 \times 10^{-12}$	$+2.14739 \times 10^{-12}$	$+2.15271 \times 10^{-12}$
0	2	4	-5.86015×10^{-15}	-1.49805×10^{-14}	-1.50419×10^{-14}
0	3	0	$+4.01117 \times 10^{-11}$	$+2.48433 \times 10^{-11}$	$+2.48674 \times 10^{-11}$
0	3	1	-1.60177×10^{-12}	-2.5426×10^{-12}	-2.54489×10^{-12}
0	3	2	$+8.41646 \times 10^{-15}$	$+4.69605 \times 10^{-14}$	$+4.70247 \times 10^{-14}$
1	0	0		-0.000374879	
1	0	1		-0.0000614364	
1	0	2		$+4.68875 \times 10^{-7}$	
1	1	0		-0.00000518082	
1	1	1		$+1.53554 \times 10^{-7}$	
1	1	2		$+9.56226 \times 10^{-10}$	
1	2	0		$+2.92378 \times 10^{-9}$	
1	2	1		$+2.13857 \times 10^{-11}$	
1	2	2		-3.87438×10^{-12}	

4.3 DISCREPANCY WITH THE TABULATED VALUES OF ENDO [1]

A small discrepancy exists between our results and those of Endo [1]. The magnitude of the difference in terms of β is at most 0.15 at high temperature and pressure. The results for high pressure, low temperature as well as those for low pressure, high temperature appear to agree well. Thus a cross term appears to be the source of the problem. However, despite numerous tests and repeated checking, the source of the discrepancy could not be traced. The checks included evaluating the sound speed derivatives, Eq. (2.1), numerically for comparison with the values obtained via analytical differentiation. The agreement was within 1 ppt for term 1, and even less for term 2.

The discrepancy is particularly unsettling, as both our and Endo's underlying expressions for c, ρ, and α are the same. However, the discrepancy is small, of the same order as the uncertainty in β caused by using the different expressions for c, ρ, and α.

5. SUMMARY

In this work three polynomial relations for β are developed: one for fresh water as a function of temperature and pressure, one for seawater with a salinity of 35 ppt as a function of temperature and pressure, and one for seawater as a function of temperature, pressure, and salinity. Also developed are a similar set of relations for the term $\Lambda \equiv \beta/(\rho c^5)^{1/2}$. Plots of the sound speed c, the density ρ, as well as β and Λ for both fresh water and seawater with a salinity of 35 ppt are presented for a wide range of temperatures and pressures. Although the precision of the polynomial fits to the underlying data is quite good (typically better than 1 ppt), the use of different underlying property data leads to uncertainities in β of about $\pm 3\%$. Due to a lack of comparison information, a similar estimate of the uncertainty is not available for β in fresh water.

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